Reconstructing $f(R)$ modified gravity from ordinary and entropy-corrected versions of the holographic and new agegraphic dark energy models

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Abstract

Here, we peruse cosmological usage of the most promising candidates of dark energy in the framework of $f(R)$ theory. We reconstruct the different $f(R)$ modified gravity models in the spatially flat FRW universe according to the ordinary and entropy-corrected versions of the holographic and new agegraphic dark energy models, which describe accelerated expansion of the universe. We also obtain the equation of state parameter of the corresponding $f(R)$-gravity models. We conclude that the holographic and new agegraphic $f(R)$-gravity models can behave like phantom or quintessence models. Whereas the equation of state parameter of the entropy-corrected models can transit from quintessence state to phantom regime as indicated by recent observations.

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1 Introduction

Astrophysical data from type Ia supernovae, cosmic microwave background radiation and large scale structure have provided convincing evidence for the present observable universe to be in the phase of accelerated expansion [1]. One explanation for the cosmic acceleration is the dark energy (DE), an exotic energy with negative pressure. The origin and nature of DE is still a mystery. A great variety of DE models have been proposed (for review see [2, 3]), but most of them are not able to explain all features of the universe, or are artificially constructed in the sense that it introduces too many free parameters to be able to fit with the experimental data.

The holographic DE (HDE) is one of interesting DE candidates which was proposed based on the holographic principle [4, 5, 6]. It was shown in [7] that in quantum field theory, the UV cut-off Λ should be related to the IR cut-off L due to limit set by forming a black hole. Following this line, Li [8] argued that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \leq LM_P^2$, where ρ_Λ is the quantum zero point energy density caused by UV cut-off Λ and M_P is the reduced Planck Mass $M_P^{-2} = 8\pi G$. Also Li [8] showed that the cosmic coincidence problem can be resolved by inflation in the HDE model, provided the minimal number of e-foldings [8]. The HDE models have been studied widely in the literature [9, 10, 11, 12, 13]. Obviously, in the derivation of HDE, the black hole entropy S_{BH} plays an important role. As is well known, usually, $S_{\mathrm{BH}} = A/(4G)$, where $A \sim L^2$ is the area of horizon. However, in the literature, this entropy-area relation can be modified to [14]

$$S_{\mathrm{BH}} = \frac{A}{4G} + \tilde{\alpha}\ln\frac{A}{4G} + \tilde{\beta}, \tag{1}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. These corrections can appear in the black hole entropy in loop quantum gravity (LQG) [15]. They can also be due to thermal equilibrium fluctuation, quantum fluctuation, or mass and charge fluctuations (for a good review see [15] and references therein). More recently, motivated by the corrected entropy-area relation (1) in the setup of LQG, the energy density of the entropy-corrected HDE (ECHDE) was proposed by Wei [15].

Recently, the original agegraphic dark energy (ADE) model was proposed by Cai [16]. The ADE model assumes that the observed DE comes from the spacetime and matter field fluctuations in the universe [16]. Following the line of quantum fluctuations of spacetime, Karolyhazy et al. [17] discussed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t \sim t_P^{2/3}t^{1/3}$ where t_P is the reduced Planck time. Based on Karolyhazy relation, Maziashvili [18] discussed that the energy density of metric fluctuations of the Minkowski spacetime is given by $\rho_\Lambda \sim 1/(t_P^2t^2) \sim M_P^2/t^2$. Based on Karolyhazy relation [17] and Maziashvili arguments [18], Cai proposed the original ADE model to explain the accelerated expansion of the universe [16]. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, the new ADE (NADE) model was proposed by Wei & Cai [19], while the time scale was chosen to be the conformal time instead of the age of the universe. It was found that the coincidence problem could be solved naturally in the NADE model [20]. The ADE models have given rise to a lot of interest recently and have been examined and studied in ample detail in [21, 22, 23, 24]. More recently, very similar to the ECHDE model, the energy density of the entropy-corrected NADE (ECNADE) was proposed by Wei [15] and investigated in ample detail by [25].

Another explanation for the cosmic acceleration and what we get from observational data is the different approach, so called "modified gravity". This scheme possesses the relevant feature that experimental data turn out to be naturally interpreted without the need of additional

components like DE (for review see [26]). In the situation when Einstein general relativity (GR) cannot naturally describe the DE epoch of the universe the search of alternative, modified gravity which is consistent with solar system tests/observational data is of primary interest [27]. A very popular modified gravity model is the so-called $f(R)$-gravity [28, 29, 30] where $f(R)$ is an arbitrary function of the scalar curvature R. Actually, among extended theories of modified gravity, $f(R)$-gravity represents a viable alternative to DE and naturally gives rise to accelerating singularity-free solution in early and late cosmic epochs [31]. For instance, the form $f(R) = R + R^2 + 1/R$ can predict the unification of the early-time inflation and late-time cosmic acceleration in the standard metric formulation [32].

Viewing the modified $f(R)$-gravity model as an effective description of the underlying theory of DE, and considering the ordinary and entropy-corrected versions of the HDE and NADE scenarios as pointing in the direction of the underlying theory of DE, it is interesting to study how the $f(R)$-gravity can describe the HDE, ECHDE, NADE and ECNADE densities as effective theories of DE models. This motivated us to establish the different models of $f(R)$-gravity according to the ordinary and entropy-corrected versions of the HDE and NADE scenarios. This paper is organized as follows. In section 2, we review the theory of $f(R)$-gravity in the metric formalism. In sections 3 to 7 we reconstruct the different $f(R)$-gravity models corresponding to the HDE, ECHDE, NADE and ECNADE models, respectively. Section 8 is devoted to our conclusions.

2 The theory of $f(R)$ modified gravity

The general $f(R)$-gravity action is given by [27, 28]

$$S = \int \sqrt{-g} \, \mathrm{d}^4x \left[\frac{R + f(R)}{2k^2} + L_{\mathrm{matter}} \right], \tag{2}$$

where $k^2 = M_P^{-2} = 8\pi G$ and $\hbar = c = 1$. Also G, g, R and L_{matter} are the gravitational constant, the determinant of metric $g_{\mu\nu}$, the Ricci scalar and the lagrangian density of the matter inside the universe, respectively.

One notes that there are in fact two strategies in $f(R)$ theories: the metric formalism, where the action is varied with respect to the metric only; and the Palatini formalism [33], where the metric and the connection are treated as two independent variables with respect to which the action is varied [34]. It is only in Einstein gravity $f(R) = 0$ that both approaches reach the same result. In general $f(R)$ theories, the problem of which approach should be used is still an open question and the final solution may be determined by further observations and theoretical development [34]. Here like [34] we use the metric formalism.

Taking the variation of the action (2) with respect to the metric $g_{\mu\nu}$, one can obtain the field equations as [27, 28]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k^2 \left(T_{\mu\nu}^{(R)} + T_{\mu\nu}^{(m)} \right), \tag{3}$$

where

$$k^2 T_{\mu\nu}^{(R)} = \frac{1}{2} g_{\mu\nu} f(R) - R_{\mu\nu} f'(R) + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) f'(R). \tag{4}$$

Here $R_{\mu\nu}$ and $T_{\mu\nu}^{(m)}$ are the Ricci tensor and the energy-momentum tensor of the matter, respectively. Also the prime denotes a derivative with respect to R.

Now if we consider the spatially flat FRW metric for the universe as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t) \sum_{i=1}^{3} (\mathrm{d}x^i)^2, \tag{5}$$

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and taking $T_\nu^{\mu(m)} = \text{diag}(-\rho_m, p_m, p_m, p_m)$ for the energy-momentum tensor of the matter in the prefect fluid form, then the set of field equations (3) reduce to the modified Friedmann equations in the framework of $f(R)$-gravity as

$$\frac{3}{k^2}H^2 = \rho_m + \rho_R, \tag{6}$$

$$\frac{1}{k^2}(2\dot{H} + 3H^2) = -(p_m + p_R), \tag{7}$$

where

$$\rho_R = \frac{1}{k^2}\left[-\frac{1}{2}f(R) + 3(\dot{H} + H^2)f'(R) - 18(4H^2\dot{H} + H\ddot{H})f''(R)\right], \tag{8}$$

$$p_R = \frac{1}{k^2}\left[\frac{1}{2}f(R) - (\dot{H} + 3H^2)f'(R)\right.$$
$$\left. +6(8H^2\dot{H} + 6H\ddot{H} + 4\dot{H}^2 + \dddot{H})f''(R) + 36(\ddot{H} + 4H\dot{H})^2f'''(R)\right], \tag{9}$$

and

$$R = 6(\dot{H} + 2H^2). \tag{10}$$

Here $H = \dot{a}/a$ is the Hubble parameter and the dot denotes a derivative with respect to cosmic time t. Also ρ_R and p_R are the curvature contribution to the energy density and pressure.

The energy conservation laws are still given by

$$\dot{\rho_m} + 3H(\rho_m + p_m) = 0, \tag{11}$$

$$\dot{\rho}_R + 3H(\rho_R + p_R) = 0. \tag{12}$$

In the case of $f(R) = 0$, from Eqs. (8) and (9) we have $\rho_R = 0$ and $p_R = 0$. Therefore Eqs. (6) and (7) transform to the usual Friedmann equations in GR.

The equation of state (EoS) parameter due to the curvature contribution is defined as [35]

$$\omega_R = \frac{p_R}{\rho_R}$$
$$= -1 - \frac{4\left[\dot{H}f'(R) + 3(3H\ddot{H} - 4H^2\dot{H} + 4\dot{H}^2 + \dddot{H})f''(R) + 18(\ddot{H} + 4H\dot{H})^2f'''(R)\right]}{\left[f(R) - 6(\dot{H} + H^2)f'(R) + 36(4H^2\dot{H} + H\ddot{H})f''(R)\right]}. \tag{13}$$

Note that for a $f(R)$ dominated universe, Eq. (6) yields

$$\frac{3}{k^2}H^2 = \rho_R. \tag{14}$$

Taking time derivative of the above relation and using the continuity equation (12), one can get the EoS parameter as

$$\omega_R = -1 - \frac{2\dot{H}}{3H^2}, \tag{15}$$

which shows that for the phantom, $\omega_R < -1$, and quintessence, $\omega_R > -1$, dominated universe, we need to have $\dot{H} > 0$ and $\dot{H} < 0$, respectively.

For a given $a = a(t)$, by the help of Eqs. (8) and (9) one can reconstruct the $f(R)$-gravity according to any DE model given by the EoS $p_R = p_R(\rho_R)$ or $\rho_R = \rho_R(a)$. There are two classes

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of scale factors which usually people consider them for describing the accelerating universe in $f(R)$, $f(\mathcal{G})$ and $f(R,\mathcal{G})$ modified gravities [36].

The first class of scale factor is given by [36, 37]

$$a(t) = a_0(t_s - t)^{-h}, \quad t \le t_s, \quad h > 0. \tag{16}$$

Using Eqs. (10) and (16) one can obtain

$$H = \frac{h}{t_s - t} = \left[\frac{h}{6(2h+1)}R\right]^{1/2}, \quad \dot{H} = H^2/h, \tag{17}$$

which $\dot{H} = H^2/h > 0$ shows that the model (16) is correspondence to a phantom dominated universe. This is why that in the literature the model (16) is usually so-called the phantom scale factor.

For the second class of scale factor defined as [36]

$$a(t) = a_0 t^h, \quad h > 0, \tag{18}$$

one can obtain

$$H = \frac{h}{t} = \left[\frac{h}{6(2h-1)}R\right]^{1/2}, \quad \dot{H} = -H^2/h, \tag{19}$$

which $\dot{H} = -H^2/h < 0$ reveals that the model (18) describes a quintessence dominated universe. Hence, the model (18) is so-called the quintessence scale factor in the literature.

In sections 3 to 6 using the two classes of scale factors (16) and (18), we reconstruct the different $f(R)$-gravities according to the HDE, ECHDE, NADE and ECNADE models.

3 $f(R)$ reconstruction from HDE model

Here we reconstruct the $f(R)$-gravity according to the HDE scenario. Following Li [8] the HDE density in a spatially flat universe is given by

$$\rho_\Lambda = \frac{3c^2}{k^2 R_h^2}, \tag{20}$$

where c is a numerical constant. Recent observational data, which have been used to constrain the HDE model, show that for the flat universe $c = 0.818^{+0.113}_{-0.097}$ [38]. Also R_h is the future event horizon defined as

$$R_h = a \int_t^\infty \frac{\mathrm{d}t}{a} = a \int_a^\infty \frac{\mathrm{d}a}{Ha^2}. \tag{21}$$

For the first class of scale factor (16) and using Eq. (17), the future event horizon R_h yields

$$R_h = a \int_t^{t_s} \frac{\mathrm{d}t}{a} = \frac{t_s - t}{h + 1} = \frac{1}{h+1}\sqrt{\frac{6h(2h+1)}{R}}. \tag{22}$$

Replacing Eq. (22) into (20) one can get

$$\rho_\Lambda = \frac{c^2(h+1)^2}{2k^2 h(2h+1)}R. \tag{23}$$

Substituting Eq. (23) in the differential equation (8), i.e. $\rho_R = \rho_\Lambda$, gives the following solution

$$f(R) = \lambda_+ R^{m+} + \lambda_- R^{m-} + \gamma_c\, R, \tag{24}$$

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where

$$m_\pm = \frac{3 + h \pm \sqrt{h^2 - 10h + 1}}{4}, \tag{25}$$

and

$$\gamma_c = -\frac{c^2(h+1)^2}{h^2}. \tag{26}$$

Also $\lambda_\pm$ are the integration constants that can be determined from the necessary boundary conditions. Following [39] the accelerating expansion in the present universe could be generated, if one consider that $f(R)$ could be a small constant at present universe, that is

$$f(R_0) = -2R_0, \tag{27}$$

$$f'(R_0) \sim 0, \tag{28}$$

where $R_0 \sim (10^{-33}\text{eV})^2$ is the current curvature. Applying the above boundary conditions to the solution (24) one can obtain

$$\lambda_+ = \frac{\gamma_c(m_- - 1) + 2m_-}{(m_+ - m_-)R_0^{m_+-1}}, \tag{29}$$

$$\lambda_- = \frac{\gamma_c(m_+ - 1) + 2m_+}{(m_- - m_+)R_0^{m_--1}}. \tag{30}$$

Replacing Eq. (24) into (13) and using (17) one can get the EoS parameter of the holographic $f(R)$-gravity model as

$$\omega_R = -1 - \frac{2}{3h}, \quad h > 0, \tag{31}$$

which corresponds to a phantom accelerating universe, i.e. $\omega_R < -1$. Recent observational data indicates that the EoS parameter ω_R at the present lies in a narrow strip around $\omega_R = -1$ and is quite consistent with being below this value [3].

For the second class of scale factor (18), using Eq. (19) the future event horizon R_h reduces to

$$R_h = a \int_t^\infty \frac{\mathrm{d}t}{a} = \frac{t}{h-1} = \frac{1}{h-1}\sqrt{\frac{6h(2h-1)}{R}}, \quad h > 1, \tag{32}$$

where the condition $h > 1$ is obtained due to have a finite future event horizon. If we repeat the above calculations then one can obtain the both of $f(R)$ and ω_R corresponding to the HDE for the second class of scale factor (18). The result for $f(R)$ is same as (24) where now

$$m_\pm = \frac{3 - h \pm \sqrt{h^2 + 10h + 1}}{4}, \tag{33}$$

$$\gamma_c = -\frac{c^2(h-1)^2}{h^2}. \tag{34}$$

Also the EoS parameter is obtained as

$$\omega_R = -1 + \frac{2}{3h}, \quad h > 1, \tag{35}$$

which describes an accelerating universe with the quintessence EoS parameter, i.e. $-1 < \omega_R < -1/3$.

4 $f(R)$ reconstruction from ECHDE model

Using the corrected entropy-area relation (1), the energy density of the ECHDE can be obtained as [15]

$$\rho_\Lambda = \frac{3c^2}{k^2 R_h^2} + \frac{\alpha}{R_h^4} \ln\left(\frac{R_h^2}{k^2}\right) + \frac{\beta}{R_h^4}, \tag{36}$$

where α and β are dimensionless constants of order unity. In the special case $\alpha = \beta = 0$, the above equation yields the well-known HDE density (20). Since the last two terms in Eq. (36) can be comparable to the first term only when R_h is very small, the corrections make sense only at the early stage of the universe. When the universe becomes large, ECHDE reduces to the ordinary HDE [15].

For the first class of scale factor (16), substituting Eq. (22) into (36) yields

$$\rho_\Lambda = \frac{c^2(h+1)^2}{2k^2 h(2h+1)} R + \frac{(h+1)^4}{36h^2(2h+1)^2}\left[\alpha \ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R}\right) + \beta\right] R^2. \tag{37}$$

Solving the differential equation (8) for the energy density (37) gives the entropy-corrected holographic $f(R)$-gravity as

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \gamma_c R$$
$$+ \frac{k^2(h+1)^4}{54h^2(2h+1)}\left\{\alpha\left[\left(\frac{h-5}{3}\right) - \ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R}\right)\right] - \beta\right\} R^2, \tag{38}$$

where $m_\pm$ and γ_c are given by Eqs. (25) and (26), respectively. Also $\lambda_\pm$ are determined from the boundary conditions (27) and (28) as

$$\lambda_+ = \frac{\gamma_c(m_- - 1) + 2m_-}{(m_+ - m_-)R_0^{m_+-1}} + \frac{k^2(h+1)^4(m_- - 2)}{54h^2(2h+1)(m_+ - m_-)R_0^{m_+-2}} \times$$
$$\left\{\alpha\left[\frac{h-5}{3} - \frac{1}{m_--2} - \ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R_0}\right)\right] - \beta\right\}, \tag{39}$$

$$\lambda_- = \frac{\gamma_c(m_+ - 1) + 2m_+}{(m_- - m_+)R_0^{m_--1}} + \frac{k^2(h+1)^4(m_+ - 2)}{54h^2(2h+1)(m_- - m_+)R_0^{m_--2}} \times$$
$$\left\{\alpha\left[\frac{h-5}{3} - \frac{1}{m_+-2} - \ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R_0}\right)\right] - \beta\right\}. \tag{40}$$

Substituting Eq. (38) into (13) one can get the EoS parameter of the entropy-corrected holographic $f(R)$-gravity model as

$$\omega_R = -1 - \frac{2}{3h}\left\{1 + \frac{k^2(h+1)^2\left[-\alpha + \alpha\ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R}\right) + \beta\right] R}{18c^2 h(2h+1) + k^2(h+1)^2\left[\alpha\ln\left(\frac{6h(2h+1)}{k^2(h+1)^2 R}\right) + \beta\right] R}\right\}, \quad h > 0, \tag{41}$$

which can be rewritten using the first relation of Eq. (17) as

$$\omega_R = -1 - \frac{2}{3h}\left\{1 + \frac{-\alpha + 2\alpha\ln\left(\frac{h}{k(h+1)H}\right) + \beta}{3c^2\left(\frac{h}{k(h+1)H}\right)^2 + 2\alpha\ln\left(\frac{h}{k(h+1)H}\right) + \beta}\right\}, \quad h > 0. \tag{42}$$

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The above relation shows that the EoS parameter is time-dependent and in contrast with constant EoS parameter (31), it can justify the transition from quintessence state, $\omega_R > -1$, to the phantom regime, $\omega_R < -1$, as indicated by recent observations [40]. Note that if we set $\alpha = \beta = 0$ then Eqs. (38) and (42) reduce to (24) and (31), respectively.

For the second class of scale factor (18), the result of $f(R)$ is obtained as

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \gamma_c R$$
$$+ \frac{k^2(h-1)^4}{54h^2(2h-1)} \left\{ \alpha \left[\left(\frac{h+5}{3} \right) + \ln \left(\frac{6h(2h-1)}{k^2(h-1)^2 R} \right) \right] + \beta \right\} R^2, \tag{43}$$

where

$$\lambda_+ = \frac{\gamma_c(m_- - 1) + 2m_-}{(m_+ - m_-)R_0^{m_+-1}} + \frac{k^2(h-1)^4(m_- - 2)}{54h^2(2h-1)(m_+ - m_-)R_0^{m_+-2}} \times$$
$$\left\{ \alpha \left[\frac{h+5}{3} + \frac{1}{m_- - 2} + \ln \left(\frac{6h(2h-1)}{k^2(h-1)^2 R_0} \right) \right] + \beta \right\}, \tag{44}$$

$$\lambda_- = \frac{\gamma_c(m_+ - 1) + 2m_+}{(m_- - m_+)R_0^{m_--1}} + \frac{k^2(h-1)^4(m_+ - 2)}{54h^2(2h-1)(m_- - m_+)R_0^{m_--2}} \times$$
$$\left\{ \alpha \left[\frac{h+5}{3} + \frac{1}{m_+ - 2} + \ln \left(\frac{6h(2h-1)}{k^2(h-1)^2 R_0} \right) \right] + \beta \right\}, \tag{45}$$

and the parameters $m_\pm$ and γ_c are given by Eqs. (33) and (34), respectively. Also the EoS parameter is obtained as

$$\omega_R = -1 + \frac{2}{3h} \left\{ 1 + \frac{-\alpha + 2\alpha \ln \left(\frac{h}{k(h-1)H} \right) + \beta}{3c^2 \left(\frac{h}{k(h-1)H} \right)^2 + 2\alpha \ln \left(\frac{h}{k(h-1)H} \right) + \beta} \right\}, \quad h > 1. \tag{46}$$

Here also to have a finite R_h, the parameter h should be in the range of $h > 1$. One notes that the EoS parameter (46) is also dynamical and in contrast with constant EoS parameter (35), it can accommodate the transition from $\omega_R > -1$ to $\omega_R < -1$ at recent stage.

5 $f(R)$ reconstruction from NADE model

Following [19], the energy density of the NADE is given by

$$\rho_\Lambda = \frac{3n^2}{k^2\eta^2}, \tag{47}$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density is derived for Minkowski spacetime), and so on. It was found that the coincidence problem could be solved naturally in the NADE model provided that the single model parameter n is of order unity [20]. The joint analysis of the astronomical data for the NADE model in flat universe gives the best-fit value (with 1σ uncertainty) $n = 2.716^{+0.111}_{-0.109}$ [20]. Also η is the conformal time of FRW universe, and given by

$$\eta = \int \frac{\mathrm{d}t}{a} = \int \frac{\mathrm{d}a}{Ha^2}. \tag{48}$$

For the first class of scale factor (16), the conformal time η by the help of Eq. (17) yields

$$\eta = \int_t^{t_s} \frac{dt}{a} = \frac{(t_s - t)^{h+1}}{a_0(h+1)} = \frac{1}{a_0(h+1)} \left[\frac{6h(2h+1)}{R} \right]^{\frac{h+1}{2}}. \tag{49}$$

Substituting Eq. (49) into (47) one can obtain

$$\rho_\Lambda = \frac{3n^2 a_0^2 (h+1)^2}{k^2 (6h(2h+1))^{h+1}} R^{h+1}. \tag{50}$$

Solving the differential equation (8) for the energy density (50) reduces to

$$f(R) = \lambda_+ R^{m+} + \lambda_- R^{m-} + \gamma_n R^{h+1}, \tag{51}$$

where

$$\gamma_n = -\frac{n^2 a_0^2 (h+1)^2}{h^2 (h+2)(6h(2h+1))^h}, \tag{52}$$

and $m_\pm$ are given by Eq. (25). Using the boundary conditions (27) and (28) the parameters $\lambda_\pm$ are determined as

$$\lambda_+ = \frac{\gamma_n(m_- - (h+1))R_0^h + 2m_-}{(m_+ - m_-)R_0^{m_+ - 1}}, \tag{53}$$

$$\lambda_- = \frac{\gamma_n(m_+ - (h+1))R_0^h + 2m_+}{(m_- - m_+)R_0^{m_- - 1}}. \tag{54}$$

Replacing Eq. (51) into (13) one can get the EoS parameter of the new agegraphic $f(R)$-gravity model as

$$\omega_R = -1 - \frac{2(h+1)}{3h}, \quad h > 0, \tag{55}$$

which like the EoS parameter of the holographic $f(R)$-gravity model (31), it always crosses the phantom-divide line, i.e. $\omega_R < -1$.

For the second class of scale factor (18), using (19) the conformal time η is obtained as

$$\eta = \int_0^t \frac{dt}{a} = \frac{t^{1-h}}{a_0(1-h)} = \frac{1}{a_0(1-h)} \left[\frac{6h(2h-1)}{R} \right]^{\frac{1-h}{2}}, \quad \frac{1}{2} < h < 1, \tag{56}$$

where the condition $\frac{1}{2} < h < 1$ is necessary due to have a real finite conformal time. The result of $f(R)$ is

$$f(R) = \lambda_+ R^{m+} + \lambda_- R^{m-} + \gamma_n R^{1-h}, \tag{57}$$

where

$$\lambda_+ = \frac{\gamma_n(m_- + h - 1)R_0^{-h} + 2m_-}{(m_+ - m_-)R_0^{m_+ - 1}}, \tag{58}$$

$$\lambda_- = \frac{\gamma_n(m_+ + h - 1)R_0^{-h} + 2m_+}{(m_- - m_+)R_0^{m_- - 1}}, \tag{59}$$

with

$$\gamma_n = \frac{n^2 a_0^2 (h-1)^2}{h^2 (h-2)(6h(2h-1))^{-h}}, \tag{60}$$

and the parameters $m_\pm$ are given by Eq. (33).

Also the EoS parameter is obtained as

$$\omega_R = -1 + \frac{2(1-h)}{3h}, \quad \frac{1}{2} < h < 1, \tag{61}$$

which shows a quintessence-like EoS parameter with $-1 < \omega_R < -1/3$.

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6 $f(R)$ reconstruction from ECNADE model

With the help of quantum corrections to the entropy-area relation (1) in the setup of LQG, the energy density of the ECNADE is given by [15]

$$\rho_\Lambda = \frac{3n^2}{k^2\eta^2} + \frac{\alpha}{\eta^4}\ln\left(\frac{\eta^2}{k^2}\right) + \frac{\beta}{\eta^4}, \tag{62}$$

which closely mimics to that of ECHDE density (36) and R_h is replaced with the conformal time η. Here α and β are dimensionless constants of order unity. In the special case $\alpha = \beta = 0$, the above equation yields the well-known NADE density (47).

For the first class of scale factor (16), substituting Eq. (49) into (62) one can get

$$\rho_\Lambda = \frac{3n^2 a_0^2 (h+1)^2}{k^2 (6h(2h+1))^{h+1}} R^{h+1}$$
$$+ \frac{a_0^4 (h+1)^4}{(6h(2h+1))^{2h+2}}\left[\alpha\ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R^{h+1}}\right) + \beta\right] R^{2h+2}. \tag{63}$$

Solving the differential equation (8) for the energy density (63) yields

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \gamma_n\, R^{h+1} - \frac{k^2 a_0^4 (h+1)^4}{3h(3+10h+6h^2)(6h(2h+1))^{2h+1}} \times$$
$$\left\{\alpha\left[\frac{(7h+5)(h+1)}{3+10h+6h^2} + \ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R^{h+1}}\right)\right] + \beta\right\} R^{2h+2}, \tag{64}$$

where $m_\pm$ and γ_n are given by Eqs. (25) and (52), respectively. Also $\lambda_\pm$ are determined from the boundary conditions (27) and (28) as

$$\lambda_+ = \frac{\gamma_n(m_- - (h+1))R_0^h + 2m_-}{(m_+ - m_-)R_0^{m_+ - 1}}$$
$$+ \frac{k^2 a_0^4 (2h+2-m_-)(h+1)^4 R_0^{2h+2-m_+}}{3h(3+10h+6h^2)(6h(2h+1))^{2h+1}(m_+ - m_-)} \times$$
$$\left\{\alpha\left[\frac{(7h+5)(h+1)}{3+10h+6h^2} + \frac{h+1}{m_- - 2 - 2h} + \ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R_0^{h+1}}\right)\right] + \beta\right\}, \tag{65}$$

$$\lambda_- = \frac{\gamma_n(m_+ - (h+1))R_0^h + 2m_+}{(m_- - m_+)R_0^{m_- - 1}}$$
$$+ \frac{k^2 a_0^4 (2h+2-m_+)(h+1)^4 R_0^{2h+2-m_-}}{3h(3+10h+6h^2)(6h(2h+1))^{2h+1}(m_- - m_+)} \times$$
$$\left\{\alpha\left[\frac{(7h+5)(h+1)}{3+10h+6h^2} + \frac{h+1}{m_+ - 2 - 2h} + \ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R_0^{h+1}}\right)\right] + \beta\right\}. \tag{66}$$

Replacing Eq. (64) into (13) yields the EoS parameter of the entropy-corrected new agegraphic $f(R)$-gravity model as

$$\omega_R = -1 - \frac{2(h+1)}{3h}\left\{1 + \frac{-\alpha + \alpha\ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R^{h+1}}\right) + \beta}{\frac{3n^2}{k^2 a_0^2 (h+1)^2}\left(\frac{6h(2h+1)}{R}\right)^{h+1} + \alpha\ln\left(\frac{(6h(2h+1))^{h+1}}{k^2 a_0^2 (h+1)^2 R^{h+1}}\right) + \beta}\right\}, \quad h > 0. \tag{67}$$

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It can be rewritten using the first relation of Eq. (17) as

$$\omega_R = -1 - \frac{2(h+1)}{3h}\left\{1 + \frac{-\alpha + 2\alpha \ln\left(\frac{1}{ka_0(h+1)}(\frac{h}{H})^{h+1}\right) + \beta}{3n^2\left(\frac{1}{ka_0(h+1)}(\frac{h}{H})^{h+1}\right)^2 + 2\alpha \ln\left(\frac{1}{ka_0(h+1)}(\frac{h}{H})^{h+1}\right) + \beta}\right\}, \quad h > 0, \quad (68)$$

which is time-dependent and in contrast with constant EoS parameter (55), it can justify the transition from $\omega_R > -1$ to $\omega_R < -1$. Note that if we set $\alpha = \beta = 0$ then Eqs. (64) and (68) reduce to (51) and (55), respectively.

For the second class of scale factor (18), the result of $f(R)$ is obtained as

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \gamma_n\ R^{1-h} + \frac{k^2 a_0^4 (h-1)^4}{3h(3 - 10h + 6h^2)(6h(2h-1))^{1-2h}} \times$$
$$\left\{\alpha\left[\frac{(7h-5)(h-1)}{3-10h+6h^2} + \ln\left(\frac{(6h(2h-1))^{1-h}}{k^2 a_0^2 (h-1)^2 R^{1-h}}\right)\right] + \beta\right\} R^{2-2h}, \quad (69)$$

where

$$\lambda_+ = \frac{\gamma_n(m_- + (h-1))R_0^{-h} + 2m_-}{(m_+ - m_-)R_0^{m_+-1}}$$
$$- \frac{k^2 a_0^4 (2 - 2h - m_-)(h-1)^4 R_0^{2-2h-m_+}}{3h(3-10h+6h^2)(6h(2h-1))^{1-2h}(m_+ - m_-)} \times$$
$$\left\{\alpha\left[\frac{(7h-5)(h-1)}{3-10h+6h^2} + \frac{1-h}{m_- - 2 + 2h} + \ln\left(\frac{(6h(2h-1))^{1-h}}{k^2 a_0^2 (h-1)^2 R_0^{1-h}}\right)\right] + \beta\right\}, \quad (70)$$

$$\lambda_- = \frac{\gamma_n(m_+ + (h-1))R_0^{-h} + 2m_+}{(m_- - m_+)R_0^{m_--1}}$$
$$- \frac{k^2 a_0^4 (2 - 2h - m_+)(h-1)^4 R_0^{2-2h-m_-}}{3h(3-10h+6h^2)(6h(2h-1))^{1-2h}(m_- - m_+)} \times$$
$$\left\{\alpha\left[\frac{(7h-5)(h-1)}{3-10h+6h^2} + \frac{1-h}{m_+ - 2 + 2h} + \ln\left(\frac{(6h(2h-1))^{1-h}}{k^2 a_0^2 (h-1)^2 R_0^{1-h}}\right)\right] + \beta\right\}, \quad (71)$$

and the parameters $m_\pm$ and γ_n are given by Eqs. (33) and (60), respectively. Also the EoS parameter is obtained as

$$\omega_R = -1 + \frac{2(1-h)}{3h}\left\{1 + \frac{-\alpha + 2\alpha \ln\left(\frac{1}{ka_0(1-h)}(\frac{h}{H})^{1-h}\right) + \beta}{3n^2\left(\frac{1}{ka_0(1-h)}(\frac{h}{H})^{1-h}\right)^2 + 2\alpha \ln\left(\frac{1}{ka_0(1-h)}(\frac{h}{H})^{1-h}\right) + \beta}\right\}, \quad \frac{1}{2} < h < 1.$$
$$(72)$$

Here also to have a real finite conformal time η, the parameter h should be in the range of $\frac{1}{2} < h < 1$. Contrary to the constant EoS parameter (61), the dynamical EoS parameter (72) can accommodate the transition from $\omega_R > -1$ to $\omega_R < -1$ at recent stage.

7 $f(R)$ reconstruction in de Sitter space

The scale factor in de Sitter space is defined as

$$a(t) = a_0 e^{Ht}, \quad H = \text{constant}, \quad (73)$$

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which can describe the early-time inflation of the universe [36]. Using Eqs. (10) and (73) one can obtain

$$H = \left(\frac{R}{12}\right)^{1/2}. \tag{74}$$

Then Eq. (8) takes the form

$$4k^2\rho_R = -2f(R) + Rf'(R). \tag{75}$$

Also the EoS parameter (13) yields $\omega_R = -1$ which behaves like the cosmological constant.

For the scale factor (73), using Eq. (74) the future event horizon R_h reduces to

$$R_h = a \int_t^\infty \frac{\mathrm{d}t}{a} = H^{-1} = \left(\frac{R}{12}\right)^{-1/2}. \tag{76}$$

Substituting Eq. (76) into HDE density (20) one can get

$$\rho_\Lambda = \frac{c^2}{4k^2}R. \tag{77}$$

Replacing Eq. (77) in the differential equation (75), i.e. $\rho_R = \rho_\Lambda$, gives the solution

$$f(R) = \lambda R^2 - c^2 R, \tag{78}$$

where λ is an integration constant. Note that in order to generate the inflation at the early universe as in Starobinsky's model [36, 41], one may require

$$\lim_{R \to \infty} f(R) \propto R^2. \tag{79}$$

We see that the holographic $f(R)$-gravity model (78) can satisfy the requirement of having to inflation (79).

Replacing Eq. (76) into ECHDE density (36) one can obtain

$$\rho_\Lambda = \frac{c^2}{4k^2}R + \left(\frac{R}{12}\right)^2 \left[\beta + \alpha \ln\left(\frac{12}{k^2 R}\right)\right]. \tag{80}$$

Solving the differential equation (75) for the energy density (80) yields the entropy-corrected holographic $f(R)$-gravity model as

$$f(R) = \lambda R^2 - c^2 R + \left(\frac{kR}{6}\right)^2 \left\{\beta \ln(R) - \frac{\alpha}{2}\left[\ln\left(\frac{12}{k^2 R}\right)\right]^2\right\}, \tag{81}$$

which in the absence of correction terms, i.e. $\alpha = \beta = 0$, recovers the result of holographic $f(R)$-gravity model (78). As we already mentioned the correction terms in ECHDE density (80) become important in the early inflation era. Equation (81) also confirms that besides the term λR^2, the corrections make sense during the inflation when $R \to \infty$.

The conformal time η for the scale factor (73) yields

$$\eta = \int_0^t \frac{\mathrm{d}t}{a} = \frac{1}{a_0 H}\left(1 - e^{-Ht}\right). \tag{82}$$

Here to obtain $\eta = \eta(R)$ one cannot replace t by R in (82). Therefore for the scale factor (73) one cannot obtain the $f(R)$-gravity models corresponding to the NADE (47) and ECNADE (62)

densities. To avoid of this problem we set $t \to \infty$ for the upper limit of the integral (82). Hence the result yields

$$\eta = \int_0^\infty \frac{\mathrm{d}t}{a} = (a_0 H)^{-1} = \left(\frac{a_0^2 R}{12}\right)^{-1/2}.$$ (83)

Replacing Eq. (83) into NADE density (47) yields

$$\rho_\Lambda = \frac{n^2 a_0^2}{4k^2} R.$$ (84)

Substituting Eq. (84) in the differential equation (75) gives the solution

$$f(R) = \lambda R^2 - n^2 a_0^2 R,$$ (85)

where λ is an integration constant. Note that the new agegraphic $f(R)$-gravity model (85) like (78) satisfies the inflation condition (79).

Replacing Eq. (83) into ECNADE density (62) one can get

$$\rho_\Lambda = \frac{n^2 a_0^2}{4k^2} R + \left(\frac{a_0^2 R}{12}\right)^2 \left[\beta + \alpha \ln\left(\frac{12}{k^2 a_0^2 R}\right)\right].$$ (86)

Solving the differential equation (75) for the energy density (86) yields the entropy-corrected new agegraphic $f(R)$-gravity model as

$$f(R) = \lambda R^2 - n^2 a_0^2 R + \left(\frac{k a_0^2 R}{6}\right)^2 \left\{\beta \ln(R) - \frac{\alpha}{2}\left[\ln\left(\frac{12}{k^2 a_0^2 R}\right)\right]^2\right\},$$ (87)

which for $\alpha = \beta = 0$, the above result reduces to the new agegraphic $f(R)$-gravity model (85). Here also like (81), during the inflation era not only the term λR^2 but also the correction terms become considerable in Eq. (87).

8 Conclusions

Here, we considered the ordinary and entropy-corrected versions of the HDE and NADE models which are originated from some significant features of quantum gravity. The HDE is motivated from the holographic hypothesis [4, 5, 6] and the NADE is originated form uncertainty relation of quantum mechanics together with the gravitational effect in GR [17, 18]. Among various candidates to explain cosmic accelerated expansion, only HDE and NADE models are based on the entropy-area relation. However, this definition can be modified from the inclusion of quantum effects, motivated from the LQG. Hence the ECHDE and ECNADE were introduced by addition of correction terms to the energy densities of HDE and NADE, respectively [15].

We investigated the HDE, ECHDE, NADE and ECNADE in the framework of $f(R)$-gravity. Among other approaches related with a variety of DE models, a very promising approach to DE is related with the modified theories of gravity known as $f(R)$-gravity, in which DE emerges from the modification of geometry. Modified gravity gives a natural unification of the early-time inflation and late-time acceleration thanks to different role of gravitational terms relevant at small and at large curvature and may naturally describe the transition from deceleration to acceleration in the cosmological dynamics [42]. We reconstructed the different theories of modified gravity based on the $f(R)$ action in the spatially flat FRW universe for the three classes of scale factors containing i) $a = a_0(t_s - t)^{-h}$, ii) $a = a_0 t^h$ and iii) $a = a_0 e^{Ht}$ and according to

the original and entropy-corrected versions of the HDE and NADE scenarios. Furthermore, we obtained the EoS parameters of the corresponding $f(R)$-gravity models. Our calculations show that for the first class of scale factor, the EoS parameter of the holographic and new agegraphic $f(R)$-gravity models always crosses the phantom-divide line. Whereas for the second class, the EoS parameter of the mentioned models behaves like the quintessence EoS parameter. The EoS parameter of the entropy-corrected holographic and new agegraphic $f(R)$-gravity models for the both of first and second classes of scale factors can accommodate the transition from quintessence state, $\omega_R > -1$, to the phantom regime, $\omega_R < -1$, as indicated by recent observations. For the third scale factor, the EoS parameter behaves like the cosmological constant. Also the $f(R)$-gravity models corresponding to the HDE, ECHDE, NADE and ECNADE can predict the early-time inflation of the universe.

Note that although $f(R)$ theories offer a chance to explain the acceleration of the universe, they are not free of problems. As an alternative to DE for driving the late-time cosmic acceleration, the $f(R)$-gravity needs to pass the cosmological tests, including the constraints about the cosmic expansion and the cosmic structure formation. In addition, as a modified gravity theory, it needs to pass the solar system test, such as the constraints on the Brans-Dicke theory [43]. Besides, in many models of $f(R)$-gravity, including the ones called realistic, the "fine-tuning" and the "cosmic coincidence" problems which are the two well-known difficulties of the cosmological constant problems, have not been essentially solved. Although there remain possibilities to solve the coincidence problem (see e.g. [44]), the existence of small parameters in $f(R)$-gravity models is still the most important problem. In case of ΛCDM model, the scale of the cosmological constant is very small compared with the Planck scale, which is unnatural, especially from the viewpoint of the unified theory of the particle physics. Even in the realistic $f(R)$ models like Hu-Sawicki's one [45] which satisfies both cosmological and solar system tests, there appear the small parameters, which would be still unnatural.

References

[1] A.G. Riess, et al., Astron. J. **116**, 1009 (1998);
   S. Perlmutter, et al., Astrophys. J. **517**, 565 (1999);
   P. de Bernardis, et al., Nature **404**, 955 (2000);
   S. Perlmutter, et al., Astrophys. J. **598**, 102 (2003).

[2] T. Padmanabhan, Phys. Rep. **380**, 235 (2003);
   P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[3] E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).

[4] P. Horava, D. Minic, Phys. Rev. Lett. **85**, 1610 (2000);
   P. Horava, D. Minic, Phys. Rev. Lett. **509**, 138 (2001);
   S. Thomas, Phys. Rev. Lett. **89**, 081301 (2002).

[5] G. 't Hooft, gr-qc/9310026;
   L. Susskind, J. Math. Phys. **36**, 6377 (1995).

[6] W. Fischler, L. Susskind, hep-th/9806039.

[7] A. Cohen, et al., Phys. Rev. Lett. **82**, 4971 (1999).

[8] M. Li, Phys. Lett. B **603**, 1 (2004).

[9] K. Enqvist, M.S. Sloth, Phys. Rev. Lett. **93**, 221302 (2004);
   Q.G. Huang, Y. Gong, JCAP **08**, 006 (2004);
   Q.G. Huang, M. Li, JCAP **08**, 013 (2004);
   Y. Gong, Phys. Rev. D **70**, 064029 (2004).

[10] E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D **71**, 103504 (2005);
   X. Zhang, F.Q. Wu, Phys. Rev. D **72**, 043524 (2005);
   B. Guberina, R. Horvat, H. Stefancic, JCAP **05**, 001 (2005);
   J.Y. Shen, B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B **609**, 200 (2005);
   B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B **611**, 21 (2005).

[11] J.P.B. Almeida, J.G. Pereira, Phys. Lett. B **636**, 75 (2006);
   B. Guberina, R. Horvat, H. Nikolic, Phys. Lett. B **636**, 80 (2006);
   H. Li, Z.K. Guo, Y.Z. Zhang, Int. J. Mod. Phys. D **15**, 869 (2006);
   X. Zhang, Phys. Rev. D **74**, 103505 (2006);
   X. Zhang, F.Q. Wu, Phys. Rev. D **76**, 023502 (2007).

[12] L. Xu, JCAP **09**, 016 (2009);
   M. Jamil, M.U. Farooq, M.A. Rashid, Eur. Phys. J. C **61**, 471 (2009);
   M. Jamil, M.U. Farooq, Int. J. Theor. Phys. **49**, 42 (2010);
   A. Sheykhi, Class. Quantum Grav. **27**, 025007 (2010);
   A. Sheykhi, Phys. Lett. B **682**, 329 (2010).

[13] K. Karami, arXiv:1002.0431;
   K. Karami, JCAP **01**, 015 (2010);
   K. Karami, J. Fehri, Phys. Lett. B **684**, 61 (2010);
   K. Karami, J. Fehri, Int. J. Theor. Phys. **49**, 1118 (2010);
   K. Karami, A. Abdolmaleki, Phys. Scr. **81**, 055901 (2010).

[14] R. Banerjee, B.R. Majhi, Phys. Lett. B **662**, 62 (2008);
   R. Banerjee, B.R. Majhi, JHEP **06**, 095 (2008);
   R. Banerjee, S.K. Modak, JHEP **05**, 063 (2009);
   B.R. Majhi, Phys. Rev. D **79**, 044005 (2009);
   S.K. Modak, Phys. Lett. B **671**, 167 (2009);
   M. Jamil, M.U. Farooq, JCAP **03**, 001 (2010);
   H.M. Sadjadi, M. Jamil, EPL, **92**, 69001 (2010);
   S.W. Wei, Y.X. Liu, Y.Q. Wang, H. Guo, arXiv:1002.1550;
   D.A. Easson, P.H. Frampton, G.F. Smoot, arXiv:1003.1528.

[15] H. Wei, Commun. Theor. Phys. **52**, 743 (2009).

[16] R.G. Cai, Phys. Lett. B **657**, 228 (2007).

[17] F. Karolyhazy, Nuovo. Cim. A **42**, 390 (1966);
   F. Karolyhazy, A. Frenkel, B. Lukacs, in *Physics as natural Philosophy* edited by A. Shimony,
   H. Feschbach, MIT Press, Cambridge, MA, (1982);

F. Karolyhazy, A. Frenkel, B. Lukacs, in *Quantum Concepts in Space and Time* edited by R. Penrose, C.J. Isham, Clarendon Press, Oxford, (1986).

[18] M. Maziashvili, Int. J. Mod. Phys. D **16**, 1531 (2007);
M. Maziashvili, Phys. Lett. B **652**, 165 (2007).

[19] H. Wei, R.G. Cai, Phys. Lett. B **660**, 113 (2008).

[20] H. Wei, R.G. Cai, Phys. Lett. B **663**, 1 (2008).

[21] H. Wei, R.G. Cai, Eur. Phys. J. C **59**, 99 (2009).

[22] Y.W. Kim, et al., Mod. Phys. Lett. A **23**, 3049 (2008);
J.P. Wu, D.Z. Ma, Y. Ling, Phys. Lett. B **663**, 152 (2008);
J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C **54**, 303 (2008);
I.P. Neupane, Phys. Lett. B **673**, 111 (2009);
A. Sheykhi, Phys. Rev. D **81**, 023525 (2010).

[23] K.Y. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B **660**, 118 (2008).

[24] A. Sheykhi, Phys. Lett. B **680**, 113 (2009);
K. Karami, M.S. Khaledian, F. Felegary, Z. Azarmi, Phys. Lett. B **686**, 216 (2010);
K. Karami, A. Abdolmaleki, Astrophys. Space Sci. **330**, 133 (2010).

[25] K. Karami, A. Sorouri, Phys. Scr. **82**, 025901 (2010);
K. Karami, et al., Gen. Relativ. Gravit. **43**, 27 (2011);
M.U. Farooq, M.A. Rashid, M. Jamil, Int. J. Theor. Phys. **49**, 2278 (2010);
M. Malekjani, A. Khodam-Mohammadi, arXiv:1004.1017.

[26] S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002).

[27] S. Nojiri, S.D. Odintsov, Phys. Rev. D **74**, 086005 (2006).

[28] A.A. Starobinsky, Phys. Lett. B **91**, 99 (1980);
R. Kerner, Gen. Relativ. Gravit. **14**, 453 (1982);
J. Barrow, A. Ottewill, J. Phys. A **16**, 2757 (1983);
V. Faraoni, Phys. Rev. D **74**, 023529 (2006);
H.J. Schmidt, Int. J. Geom. Math. Phys. **4**, 209 (2007).

[29] S. Nojiri, S.D. Odintsov, Gen. Relativ. Gravit. **36**, 1765 (2004);
S.D. Odintsov, S. Nojiri, Mod. Phys. Lett. A **19**, 627 (2004);
M.C.B. Abdalla, S. Nojiri, S.D. Odintsov, Class. Quantum Grav. **22**, L35 (2005).

[30] S. Nojiri, S.D. Odintsov, Phys. Lett. B **576**, 5 (2003);
S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner, Phys. Rev. D **70**, 043528 (2004);
S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B **634**, 93 (2006);
M.R. Setare, Int. J. Mod. Phys. D **17**, 2219 (2008);
S. Nojiri, S.D. Odintsov, D. Sáez-Gómez, Phys. Lett. B **681**, 74 (2009);
Y. Bisabr, Phys. Scr. **80**, 045902 (2009);
A. Khodam-Mohammadi, P. Majari, M. Malekjani, Astrophys. Space Sci. **331**, 673 (2011).

[31] S. Capozziello, S. Nojiri, S.D. Odintsov, A. Troisi, Phys. Lett. B **639**, 135 (2006).

[32] S. Nojiri, S.D. Odintsov, Phys. Rev. D **68**, 123512 (2003).

[33] M. Ferraris, M. Francaviglia, I. Volovich, Class. Quantum Grav. **11**, 1505 (1994);
G. Allemandi, A. Borowiec, M. Francaviglia, Phys. Rev. D **70**, 043524 (2004);
G. Allemandi, A. Borowiec, M. Francaviglia, Phys. Rev. D **70**, 103503 (2004).

[34] X. Wu, Z.H. Zhu, Phys. Lett. B **660**, 293 (2008).

[35] K. Nozari, T. Azizi, Phys. Lett. B **680**, 205 (2009).

[36] S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).

[37] H.M. Sadjadi, Phys. Rev. D **73**, 063525 (2006).

[38] M. Li, X.D. Li, S. Wang, X. Zhang, JCAP **06**, 036 (2009).

[39] S. Nojiri, S.D. Odintsov, Phys. Lett. B **657**, 238 (2007);
S. Nojiri, S.D. Odintsov, Phys. Rev. D **77**, 026007 (2008).

[40] U. Alam, V. Sahni, A.A. Starobinsky, JCAP **06**, 008 (2004);
D. Huterer, A. Cooray, Phys. Rev. D **71**, 023506 (2005);
Y. Wang, M. Tegmark, Phys. Rev. D **71**, 103513 (2005).

[41] A. Starobinsky, JETP Lett. **86**, 157 (2007).

[42] L.N. Granda, arXiv:0812.1596.

[43] V. Faraoni, arXiv:0810.2602;
K. Henttunen, T. Multamäki, I. Vilja, Phys. Rev. D **77**, 024040 (2008);
S. Capozziello, V.F. Cardone, V. Salzano, Phys. Rev. D **78**, 063504 (2008);
T.P. Sotiriou, J. Phys. Conf. Ser. **189**, 012039 (2009);
W.T. Lin, J.A. Gu, P. Chen, arXiv:1009.3488.

[44] S. Nojiri, S.D. Odintsov, arXiv:1011.0544;
Y. Bisabr, Phys. Rev. D **82**, 124041 (2010).

[45] W. Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007).